Starting and Control Characteristics of Nuclear Rocket Engines

WILLEM JANSEN* AND JOHN K. BUCKNER†
Northern Research and Engineering Corporation, Cambridge, Mass.

The problem of the starting and control of a hypothetical nuclear rocket engine with a bleed cycle arrangement is studied parametrically with the aid of an analog computer. The study includes the investigation of both the open-loop (with a programmed control rod ramp) and controlled systems. Starts are considered with the reactor initially subcritical and critical. The open-loop system is inherently well behaved. Temperature changes are not as severe as power variations, although permissible temperature gradients may be exceeded. The initial transients of the uncontrolled system are due mainly to the time constants associated with thermal and mechanical processes, whereas the long-term transient is the result of the neutron precursor groups with long half-lives. It is possible to control the temperature and pressure satisfactorily and to meet realistic limitations on maximum design temperature and temperature rise rates. Control is accomplished by a reactor core exit pressure reference feedback control of the turbopump system flow rate and a reactor core exit gas temperature reference feedback control of the reactor power output. Controlled but rapid transitions from initial to full power are possible, and changes in thrust at design conditions are performed rapidly without excessive temperature excursions.

Nomenclature

$c_i (i = 1,2\ldots)$ = precursor concentration
c = specific heat of reactor material
c_p = specific heat of hydrogen
G_1 = gain on pressure proportional controller
G_2 = gain on pressure integral controller
G_3 = gain on temperature integral controller
h = enthalpy
I = moment of inertia
k_1 = constant of proportionality
k_2 = $\omega c_p (T_4)^{1/2}/P_4$
L = leak term in power balance equation
L' = $L/k_1 n_d$
l = average neutron lifetime
M = reactor mass
N = rotational speed
n = neutron density
P = pressure
ΔP = pressure differential defined by Eq. (23)
T = temperature
t = time
V = valve setting
V_x = desired valve setting
α_H = hydrogen density reactivity coefficient
α_H' = dimensionless hydrogen density reactivity coefficient
α_θ = bare core temperature reactivity coefficient
α_θ' = dimensionless bare core temperature reactivity
β = precursor yield fraction
$\beta_i (i = 1,2\ldots)$ = yield fraction of a particular precursor group
γ = ratio of hydrogen specific heats
$\lambda_i (i = 1,2\ldots)$ = precursor decay constant for a particular group
ρ = reactivity
$\dot{\rho}$ = time rate of change of reactivity
ρ' = dimensionless reactivity
ρ_{cr} = control-rod reactivity
ρ_x = temperature feedback control-rod reactivity
ρ_{ramp} = control-rod ramp reactivity
ρ_H = hydrogen density
T = torque
T_f = friction torque
τ_r = reactor time constant = $M_c T_{cd}/\omega_d c_p T_{4d}$
τ_p = turbopump time constant
τ_v = valve time constant
ω = mass flow rate

Superscript

(′) = dimensionless quantity

Subscripts

c = core
cr = control rod
d = design value of a variable
D = desired value of a variable
H = hydrogen
i = initial value of a variable
p = pump
t = turbine
1,2, etc. = position specified in Fig. 1

Received by ARS February 21, 1962; revision received December 17, 1962. The work described in this paper was performed under the sponsorship of the Pratt & Whitney Division of the United Aircraft Corporation. The authors wish to thank A. H. Stenning, H. P. Smith Jr., D. J. Lowen, and the staff at Pratt & Whitney Division of the United Aircraft Corporation for their assistance and suggestions during the course of this work.

* Project Engineer.
† Senior Engineer.

Introduction

THE nuclear rocket is entering a phase of intensive development because of its potential superiority over the chemical rocket for space missions requiring large payloads. Rapid and accurate control of both thrust and temperature are essential to the success of a nuclear rocket system.[1,2]

The nuclear rocket system can be divided naturally into two subsystems, the reactor and the turbopump. The former provides heat for propulsion and drives the pump-turbine. The latter delivers propellant to the reactor at high pressure. It is the interaction between these subsystems which presents the major control problem.

The starting process is initiated by opening a valve and allowing the hydrogen to flow through the system and to exert a starting torque on the pump-turbine. Then the turbine accelerates, and the temperature of the reactor increases from its initial value to some higher steady-state values; this temperature should be as high as possible to maximize the specific impulse, and the starting process should occur in the shortest possible time to minimize propellant wastage. However, material properties limit both the allowable temperature level and its time rate of change.

This study was undertaken to investigate the starting process for a bleed turbine system, with the objective of defining the type of control system which will be required to optimize the engine performance without exceeding the allowable limits.

The dynamic behavior of the engine has been studied over a wide range of the system parameters and with the reactor initially subcritical as well as critical. The system is controlled through pressure and temperature feedback to the turbine valve and the control rods, respectively.[3]

Analysis of the Reactor System

Discussion of the Reactor

The reactor considered in this study is taken to be a highly enriched, uranium-fueled, and graphite-moderated reactor with hydrogen employed as coolant and propellant. The core consists of a graphite matrix, impregnated with uranium-235, the atom ratio of carbon to uranium-235 being of the order of 500.

If a reactor is to be successful as an energy source for rocket engines, enough reactivity must be available from either control rods or hydrogen to make the reactor prompt critical under some operating conditions. Prompt criticality insures very rapid changes in power level, the reactor period being of the order of 0.01 sec, and may sometimes be desirable during start-up or acceleration. The possibility of prompt criticality demands that the core possess a built-in control system, since any mechanical device would be too slow to insure proper instantaneous control. A prompt negative temperature coefficient is such a built-in safety device in a reactor of this type. Since the introduction of hydrogen adds reactivity, it is essential to control the hydrogen density in the core. Short- and long-term control then can be achieved by a combination of the three sources of reactivity: core temperature, hydrogen density, and control rods (or other mechanical control devices). The final temperature after start-up is a result of the changes of these three sources of reactivity. A precisely calibrated engine system is required to insure that exactly the desired design temperature is reached after start-up.

In considering the dynamic behavior of a reactor of the foregoing type, a lumped parameter model is employed.[4] The consequence of this model is that one value of a parameter characterizes the value of this parameter throughout the core. The flow of hydrogen in the core is assumed to be in the gaseous state at all times. This model is approximate, but it has the advantage that it shows the fundamental relationship of the variables in a simple way. In order to obviate the importance of any inaccuracies accruing from such a model, coefficients are varied over a wide range of possible values and the system investigated parametrically.

Equations

Equations (1) and (2) are the standard neutron kinetic and precursor equations assuming no extraneous source of neutrons:

$$dn/dt = [(\rho - \beta)/l]n + \Sigma_i \lambda_i c_i \qquad (1)$$

$$dc_i/dt = (\beta_i n/l) - \lambda_i c_i \qquad (2)$$

These kinetic equations are derived from the reactor diffusion equation under a one-group neutron energy group model and Fermi-Age theory. Although strictly applicable only for well-thermalized reactors, these equations have proved to give very good results for intermediate and even fast reactors.

Delayed neutrons act as a built-in nuclear control on neutron density and power variations. For $\rho \geqslant \beta$ the reactor is prompt critical and the period becomes very small, approaching that for the case in which there are no delayed neutrons. The reactor then is not affected significantly by delayed neutrons because the reactor is critical even without delayed neutrons. Under this condition the reactor has lost its built-in nuclear control, and it is essential to introduce other kinds of control on reactivity. A negative bare core temperature coefficient and a positive hydrogen density coefficient is of extreme importance in this case for obvious reasons.

In the present study, only two precursor groups are employed, the constants being derived by approximating the exact transfer function. All six groups were simulated on the analog computer for a few cases, and it was found that the two-group simulation gives almost the same accuracy as six groups.

Because of loss of precursors and other effects, there results a net decrease in the value of β from the experimentally determined value. A value of 0.0050 for β is assumed in this study[1] to account for these effects in a conservative manner. In accordance with this precursor loss, values for the β_i's are assigned as follows:

α_θ	$= \$1.00$	P_{2d}	$= 900$ psia
α_H	$= \$2.20$	T_{4d}	$= 4000°$
β	$= 0.005$	T_4/T_H	$= 2.0$
β_1	$= 0.0025$	T_c/T_4	$= 1.0$
β_2	$= 0.0025$	τ_p	$= 0.106$
λ_1	$= 0.046$ sec^{-1}	T_f	$= \frac{3}{4}\%$ of T_{td}
λ_2	$= 0.742$ sec^{-1}		
τ_r	$= 5$ sec		
l	$= 10^{-4}$ sec		

Basic control system:

G_1	$= 10$
G_2	$= 10$
G_3	$= 0.008$
τ_{cr}	$= 0.2$ sec
τ_v	$= 0.2$ sec
P_1/P_{2d}	$= 0.035$

If the variables in Eqs. (1) and (2) are divided by their steady-state, design values, these equations can be written in dimensionless form:

$$dn'/dt = (\beta\rho' n'/l) - (\beta n'/l) + \Sigma_i(\beta_i c_i'/l) \qquad (3)$$

$$dc_i'/dt = \lambda_i n' - \lambda_i c_i' \qquad (4)$$

Equation (5) is the form of the reactivity equation employed in this study:

$$\rho = \rho_{cr} + \alpha_H(P_H/T_H) - \alpha_\theta(T_c)^{1/2} \qquad (5)$$

It is assumed in this analysis that there is enough control-rod reactivity to handle any positive or negative swing in reactivity due to hydrogen density or core temperature. For open-loop, critical starts, the control-rod reactivity ρ_{cr} is programmed as a ramp function. The initial and final values of the ramp are chosen such that the reactivity ρ is zero initially and at the final steady state. For subcritical, open-loop starts, the control-rod reactivity is held fixed at a value that would compensate for the full power, steady-state contributions from hydrogen density and core temperature.

The hydrogen density effect is the result mainly of the moderating ability of hydrogen. Increased hydrogen density decreases the fast leakage of neutrons proportionately and increases the reactivity. The negative temperature coefficient arises from the decrease in absorption with increased temperature. This results in an increase in the diffusion length and a resultant decrease in reactivity.

The coefficients α_H and $-\alpha_\theta$ are calculated from numbers found in the literature which seem representative of reactors of this type. However, these coefficients are varied widely over a possible range of values so that all possible combinations are covered.

In the present analysis it is assumed that the average reactor core temperature is proportional to the exit coolant temperature as well as to the average coolant temperature in the core. In dimensionless form, then, the following temperature relations hold:

$$T_c' = T_H' = T_4' \qquad (6)$$

where T_4' refers to the exit coolant temperature (see Fig. 1). Also, P_4 is proportional to P_H, with the result that P_4', the hydrogen outlet pressure, can be substituted for P_H' to a very good approximation. Therefore, Eq. (5) can be rewritten in dimensionless form as

$$\rho' = \rho_{cr}' + \alpha_H'(P_4'/T_4') - \alpha_\theta'(T_4')^{1/2} \qquad (7)$$

Equation (8) represents a power balance on the reactor:

$$Mc(dT_c/dt) = k_1 n - \omega c_p(T_4 - T_3) - L \qquad (8)$$

The first term on the right side of this equation represents the power released in fission. This term includes the assumption that the effective fission cross section is independent of temperature. The second term represents the heat carried away by the coolant per unit time. The third term is a constant power leak term that accounts for the initial reactor idle power that is removed by radiation. Since idle power is assumed to be a very small fraction of full design power, this leak term becomes negligible almost immediately upon start-up. The term on the left-hand side of Eq. (8) expresses the time rate of change of thermal energy of the core.

It is assumed that the flow rate ω is proportional to $P_4/T_4^{1/2}$. This assumption is valid for a space start as choking occurs in the main nozzle immediately, but it is not true initially for ground start. However, the error in this assumption for ground start is almost cancelled by the error in the core pressure drop. The temperature T_3 is very small in comparison with T_4 and can be neglected. Therefore, Eq. (8) can be rewritten as

$$Mc(dT_c/dt) = k_1 n - k_2 P_4(T_4)^{1/2} - L \qquad (9)$$

In order to put Eq. (9) in dimensionless form, it is assumed that the leak term L can be neglected at full power. Then

$$k_1 n_d = k_2 P_{4d}(T_{4d})^{1/2} = \omega_d c_p T_{4d} \qquad (10)$$

Recalling Eq. (6), Eq. (9) can be written in the dimensionless form

$$\tau_r(dT_4'/dt) = n' - P_4'(T_4')^{1/2} - L' \qquad (11)$$

Equations (3, 4, 7, and 11) define the "reactor system." The coefficients used in the solution of these equations were given previously.

These equations are all in dimensionless form, suitable for analog computation except for a scaling technique that insures that the maximum values of the variables do not exceed the range of the machine.

Analysis of the Turbomachinery

System Analysis and Dynamics

Several turbomachinery arrangements have been suggested for nuclear rockets. In this study the bleed system is used to illustrate the starting and control characteristics of nuclear rocket engines and appears in Fig. 1. A small amount of hot hydrogen is bled off from the thrust chamber and mixed with a small quantity of cold hydrogen. The mixture is fed into an axial flow turbine and is exhausted through an auxiliary nozzle, producing some additional thrust. The bleed cycle arrangement has the advantage that it is simple and fast. However, the hydrogen flow through the auxiliary nozzle represents a power loss, since the hydrogen is at a temperature considerably lower than that being ejected from the main nozzle. There are other cycle arrangements that do not have this disadvantage.

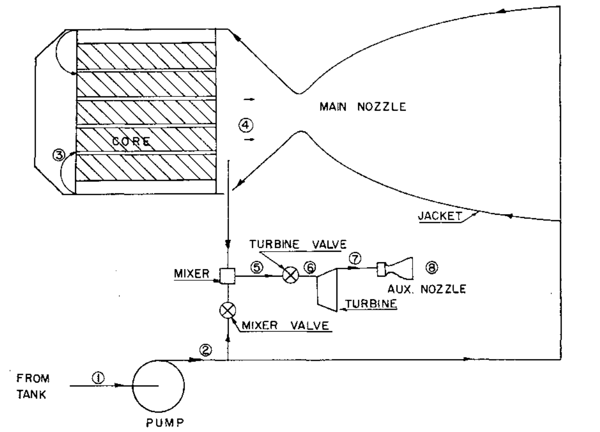

Fig. 1 Schematic diagram of nuclear rocket with bleed cycle

When the main valve is opened, the stored hydrogen enters the pump and turbine, which at that instant have no rotational speed and exchange no work. Although the hydrogen is treated as a compressible fluid, the transient effects of fluid inertia, two-phase flow, and compressibility are neglected. It is assumed that the pump is precooled to the operating temperature and also that no cavitation will occur at very small pressures. Omitting the precooling and allowing for the possibility of cavitation will slow down the starting process somewhat. The problem of the two-phase hydrogen flow initially requires a large study in itself. It can cause severe instabilities in the system, as have been experienced in the Centaur Hydrogen-Oxygen rocket. The charging times associated with the pump and turbine are less than 0.02 sec, whereas the acoustic propagation times involved are less than 0.003 sec. These constants are an order of magnitude smaller than the time constants associated with the mechanical inertia of the turbomachinery and the thermal inertia of the reactor core. For this reason, the high and low frequency transients of the fluid properties can be decoupled.[5] In this study, the low-frequency response is considered, and it is assumed that the high-frequency response has no effect on the low-frequency behavior.

For space starts the following assumptions are made:

1) The main nozzle and the turbine first-stage nozzle are assumed to be choked, or

$$\omega(T_4)^{1/2}/P_4 = \omega_d(T_{4d})^{1/2}/P_{4d} \qquad (12)$$

and

$$\omega_t(T_6)^{1/2}/P_6 = \omega_{td}(T_{6d})^{1/2}/P_{6d} \qquad (13)$$

2) For an uncontrolled system, it is assumed that the turbine valve is always full open. Then the variables with subscript 5 equal those with subscript 6.

3) The pressure loss throughout the entire system is considerable. The usual procedure for calculating the pressure loss involves the use of the friction factor. Unfortunately, this friction factor does not remain constant during the transient period. It is a function of the Reynolds number which varies during the transient period. This infers that the pressure ratio P_2'/P_4' is not constant. The change of P_2'/P_4' with P_2' has been calculated, and the results of this calculation are shown in Fig. 2. For very low values of P_2'

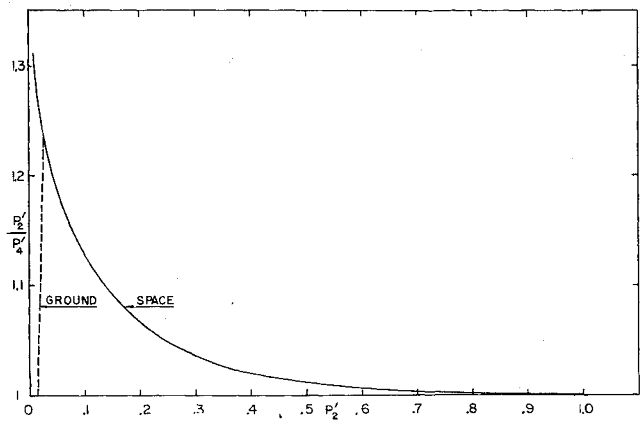

Fig. 2 Off-design pressure ratio for ground and space operations

the pressure ratio is around 1.25, whereas at design conditions this ratio is 1.00.

4) The temperature entering the turbine T_6 is assumed to be a constant fraction of the thrust chamber temperature T_4. To accomplish this, the mixer valve must be controlled during the transient period. Written in analytical form,

$$T_6 \propto T_4 \qquad T_6' = T_4' \tag{14}$$

However, the consequences of this assumption will affect the acceleration of the system only to a very small extent. The temperature T_6 in the turbine equation has only minor influence on the total torque. This suggests using a fixed setting of the mixer valve over the whole start-up sequence.

A theoretical comparison of a system having a fixed mixer valve with a system having a controlled mixer valve has been performed. The hydrogen may enter the turbine with a temperature as low as 250°R. The critical temperature of hydrogen is around 60°R, so that two-phase flow in the turbine is unlikely. Nevertheless, some experimental information is needed to determine the starting characteristics of a turbine that operates over such a wide temperature range in a short time. It is suggested that a mixer valve with two settings be used. Thus the smaller setting (nearly closed) can be used during the initial transient while the hydrogen temperature T_4 is still low. When the temperature approaches its design value and a relatively larger cold flow is required, the valve is switched to its larger setting.

For ground operation, the main nozzle and turbine first-stage nozzle become unchoked when P_2 falls below 40 psia. The auxiliary nozzle at the exhaust of the turbine becomes unchoked when P_2 is about 70 psia.‡ This affects the turbine pressure ratio and the mass flow through the turbine. The turbine torque is reduced substantially; in fact, the turbine torque decreases to zero when P_2 becomes equal to atmospheric pressure.

The calculation of the precise turbine torque under the foregoing conditions is quite involved, and consequently its representation on an analog computer is not practical. To account for the reduced turbine torque for ground conditions, a factor is calculated and multiplied by the ideal turbine torque (i.e., space torque) to obtain the "ground" torque. This multiplying factor is largely a function of P_2. Figure 3 shows the ratio of ground to space torque vs the pressure P_2.

Equations

The rate of increase of turbine-pump speed is determined by equating the inertial torque to the resultant torque. Hence,

$$I(dN/dt) = \mathrm{T}_t - \mathrm{T}_p - \mathrm{T}_f \tag{15}$$

‡ Representative turbine characteristics were used in calculating these pressures.

Representative pump and turbine characteristics furnished by the manufacturer (Pratt & Whitney) make it possible to state the following relations between the variables:

$$\mathrm{T}_t'/\omega_t'(\Delta h')^{1/2} = 1.15 - 0.15\, N'/\Delta h'^{1/2} \tag{16}$$

The enthalpy drop across the turbine can be written as

$$\Delta h = c_p[1 - (P_7/P_6)^{(\gamma-1)/\gamma}]T_6 \tag{17}$$

Assuming that the turbine pressure ratio P_7/P_6 is constant for space operation and introducing Eq. (13) into (16), it follows that

$$\mathrm{T}_t' = 1.15[P_6' - 0.13(P_6'N'/T_6'^{1/2})] \tag{18}$$

For the pump

$$(P_2 - P_1)/(P_{2d} - P_1) = N'^2 \tag{19}$$

where the tank pressure P_1 is assumed to remain constant during the starting process and

$$\mathrm{T}_p'/N'^2 = 0.06 + 0.94(\omega_p'/N') \tag{20}$$

From the assumption that the main nozzle is choked, it follows that

$$\mathrm{T}_p' = 0.06N'^2 + 0.94(P_4'N'/T_4'^{1/2}) \tag{21}$$

and

$$P_2' = [1 - (P_1/P_{2d})]N'^2 + (P_1/P_{2d}) \tag{22}$$

Since multiplication is not as accurate as addition in an analog computer, it is more convenient to obtain P_4' by deriving a certain $\Delta P'$, such that

$$P_4' = P_2' - \Delta P' \tag{23}$$

It is assumed that the pressure before the turbine valve, P_5, is equal to the pressure in the thrust chamber, P_4, and that the turbine valve is full open so that $P_6' = P_5'$. Setting the turbopump time constant IN_d/T_{td} equal to τ_p, the equations governing the dynamics of the turbomachinery become

$$\tau_p(dN'/dt) = \mathrm{T}_t' - 0.06N'^2 - 0.94(P_4'N'/T_4'^{1/2}) - (\mathrm{T}_f/\mathrm{T}_{td}) \tag{24}$$

$$\mathrm{T}_t' = 1.15[P_4' - 0.13(P_4'N'/T_4'^{1/2})] \tag{25}$$

$$P_2' = [1.0 - (P_1/P_{2d})]N'^2 + (P'/P_{2d}) \tag{26}$$

For ground level starting conditions, the torque Eq. (18) is multiplied by a factor (see Fig. 3) to take into account the effect of the reduction in torque due to the smaller pressure drop across the turbine.

A starting criterion for space operation is that dN'/dt be greater than zero in Eq. (24) at t equal to zero. A conservative value for $\mathrm{T}_f/\mathrm{T}_{td}$ for this turbopump combination is $\frac{3}{4}\%$. Then, $P_4' > 0.0065$, and it can be seen with the aid of Fig. 4 that P_2' must be greater than 0.0081 in order for start-up to occur in space. That is, the tank pressure must be at least 0.8% of the design point pump delivery pressure.

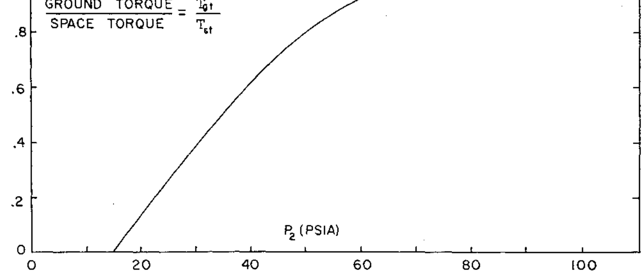

Fig. 3 Turbine torque factor for ground operation

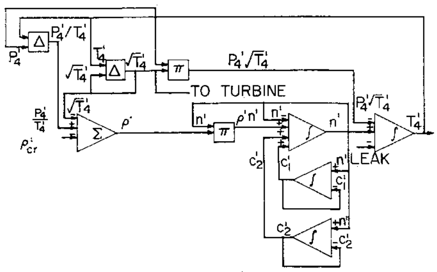

Fig. 4 Reactor analog diagram

To derive a starting criterion for ground operation, Fig. 3 again must be used to determine the actual torque. The minimum tank pressure at $t = 0$ must be at least 2.6% of P_{2d} in order to furnish enough torque to overcome the friction torque.

If the main nozzle is plugged during the first few seconds of the starting process, the maximum torque from the turbine can be obtained for ground level starts. In this case, all the hydrogen flows through the turbine initially, and the torque output of the turbine is increased.

Results for the Open-Loop System

Figure 4 shows a block diagram with the different variables for the reactor system. The turbomachinery is represented as a "black box" into which the temperature is fed and from which the pressure is obtained. Figure 5 is a block diagram for the turbomachinery subsystem. The only connections in the analytical model between the reactor and the turbomachinery are the pressure and temperature. In this program a function generator is used to represent nonlinear relationships.

Figures 6–10 show some of the results of this study for the open-loop system. The variables are shown in dimensionless units, design values corresponding to a value of unity. For the "basic" case considered in this study, the design values were given previously; P' and T' in the figures refer to P_4' and T_4' in the derived equations. In the following parametric studies, all the input values are left at the "basic" values, except for the parameter of interest.

Critical Open-Loop Response

Considering the initial and final design reactor reactivities (with the reactor critical in both cases), it is found that the control rods must supply a negative amount of reactivity. Therefore, the control-rod ramp discussed in the following is a ramp with negative slope. Figures 6–9 indicate the type of results obtained from the analog computer study for the critical, open-loop system with a programmed control-rod reactivity ramp. An initial temperature of 25% of design temperature is assumed for all critical starts. Presentation of results is limited to space starts. Except for the fact that P_1 must be greater for start-up to occur, the ground start response of the rocket is almost identical with the space start response.

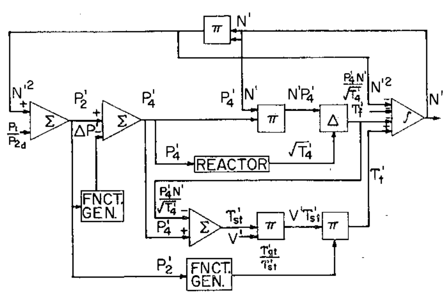

Fig. 5 Turbopump analog diagram

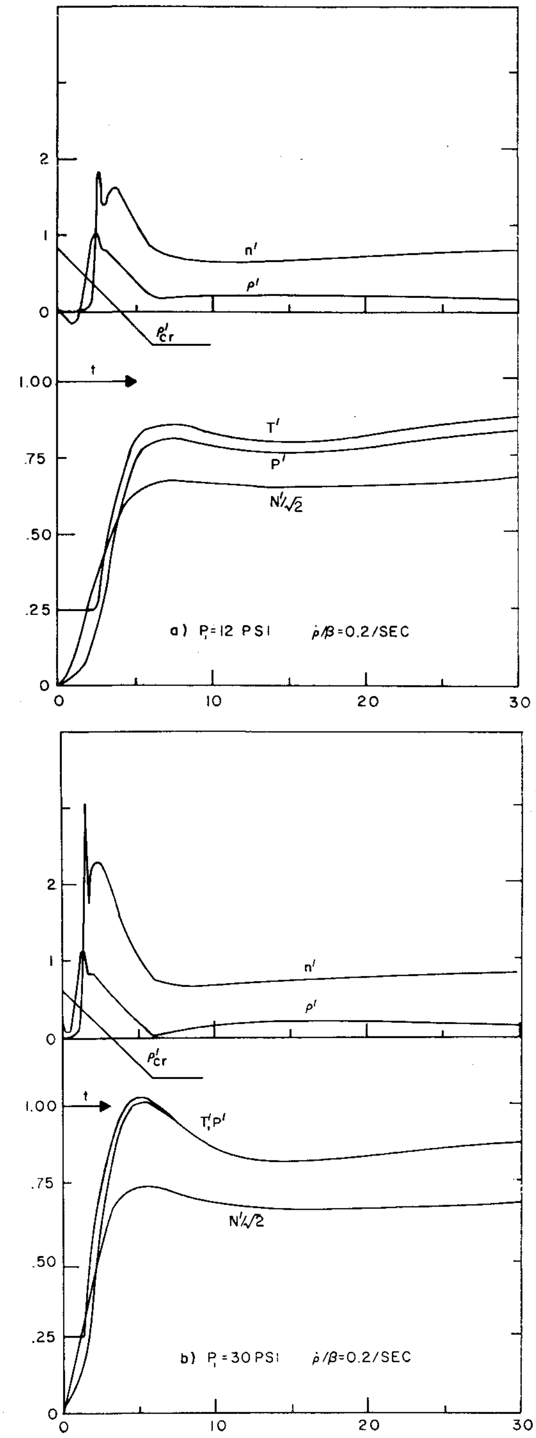

Fig. 6 Open-loop response of the system variables for space operation

In general, the initial start-up transients are determined to a great extent by the mechanical and thermal characteristics of the system, whereas the long-term behavior is due to the precursor groups with long half-lives. The resulting long reactor period is a result of the vanishingly small reactivity and is definitely the limiting time constant of the system.

ρ/β per Second as a Parameter§

There are practical limitations on control-rod worth and maneuverability. In the present study, the availability of

§ In the figures, reactivity rate is denoted by $\dot{\rho}/\beta$, or the number of β's of reactivity per second.

reactivity is limited to 0.5 β/sec. Within this limitation, the effect of this control-rod reactivity ramp speed is investigated for its effect on the open-loop, critical response.

Figures 6b and 7 indicate the effect of ρ/β/sec on the response for space start for P_1 equal to 30 psi. The reactivity ramp speed has an appreciable effect on the system response. The primary consideration is that at low ramp speeds the temperature increases very rapidly and goes through a maximum that is greater than the design value. Since the temperature at the reactor exit is the limiting design consideration, this peaking cannot be tolerated. At ramp speeds greater than about 0.3 β/sec (for $P_1 = 30$ psi), this characteristic is eliminated, although the initial response is somewhat slower. Nevertheless, the steady-state values of the variables are attained in approximately the same time regardless of the ramp speed.

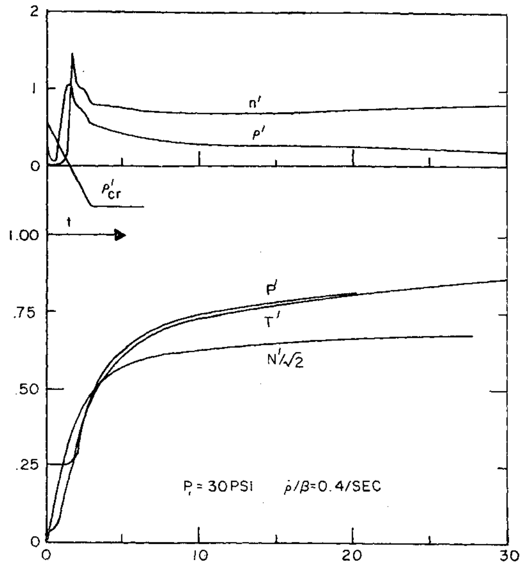

Fig. 7 Open-loop response of the system variables for space operation

Hydrogen Tank Pressure P_1 as a Parameter

The hydrogen tank pressure is an important parameter in the determination of the ability to start. Figures with the same reactivity ramp speed can be compared in order to establish the effect of a variation in P_1. For instance, Figs. 6a and 6b can be compared. For a greater P_1, P_4' begins to increase sooner. Reactivity, for a greater P_1 attains a greater positive value before going through a maximum due to the negative temperature feedback. This results in a very sharp peak in neutron density and a faster initial temperature rise. Here again this faster initial response has no effect on the long-term transient.

Also, it can be pointed out that, as P_1 is increased, a faster control-rod reactivity ramp is necessary to prevent the temperature from going through an early maximum that exceeds the design value. A ramp speed of about 0.2 β/sec is required to control temperature for P_1 equal to 12 psi, whereas approximately 0.3 β/sec is required at $P_1 = 30$ psi.

Reactor Thermal Time Constant τ_r as a Parameter

The variation of the time constant in practice probably will be the result of increased power demands on the system. In the present study, values of τ_r are varied from the basic value of 5 sec to one fifth of this value.

The variation in τ_r by a factor of 5 makes very little change in the overall system response. There is no visible change in the long-term response, but a large change is apparent in

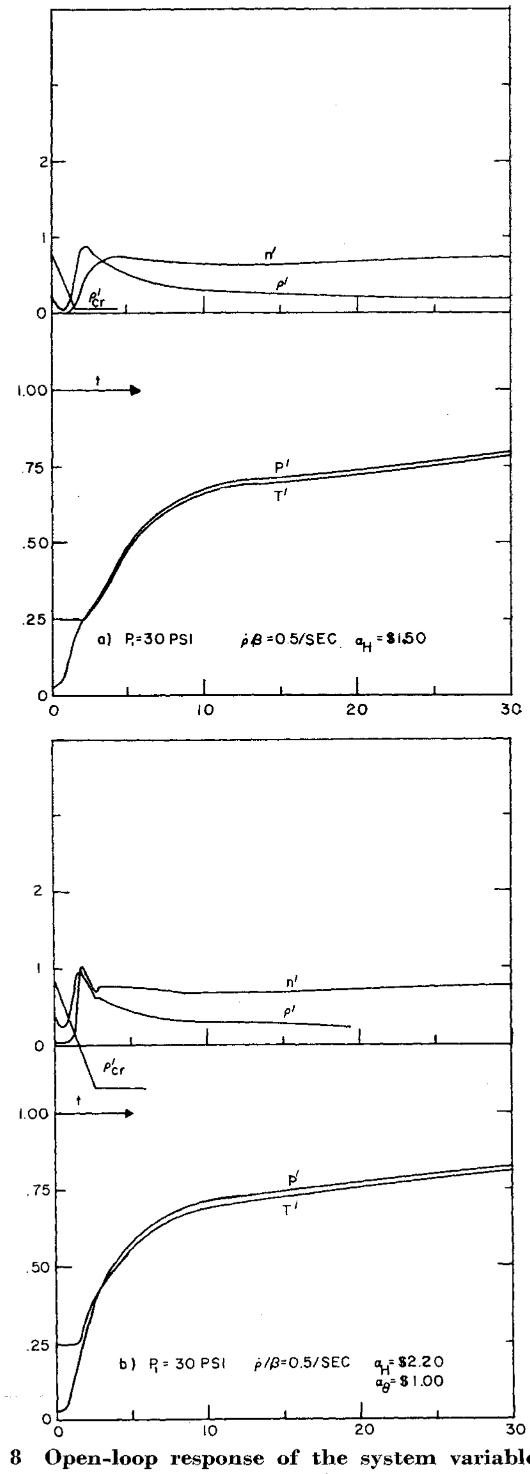

Fig. 8 Open-loop response of the system variables for space operation

the initial neutron density transient and, to a lesser extent, in the temperature transient. The initial neutron density response is slowed by decreasing the thermal time constant of the reactor. The same effect, is, of course, true for the temperature response but to a much smaller degree. The higher pressure makes the initial transient faster and in this way could be used to offset the effect of a small thermal time constant.

Hydrogen Density Coefficient α_H as a Parameter

The positive hydrogen density effect of reactivity is, of course, basic to a nuclear rocket design of the type considered here. However, a value for α_H is not calculated

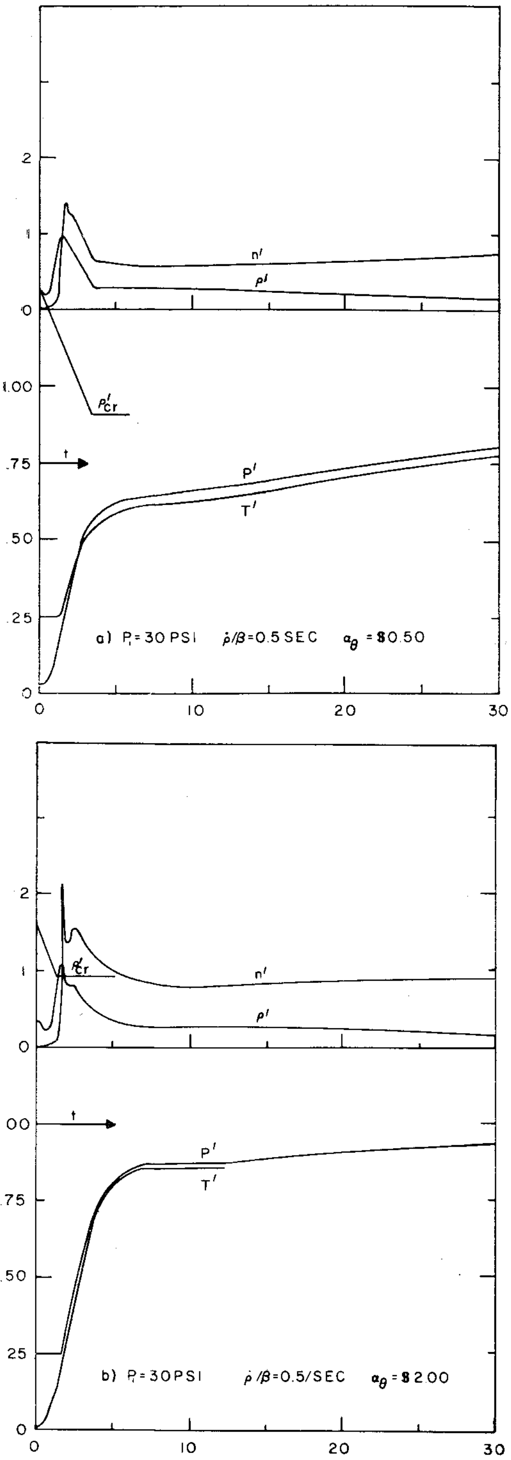

Fig. 9 Open-loop response of the system variables for space operation

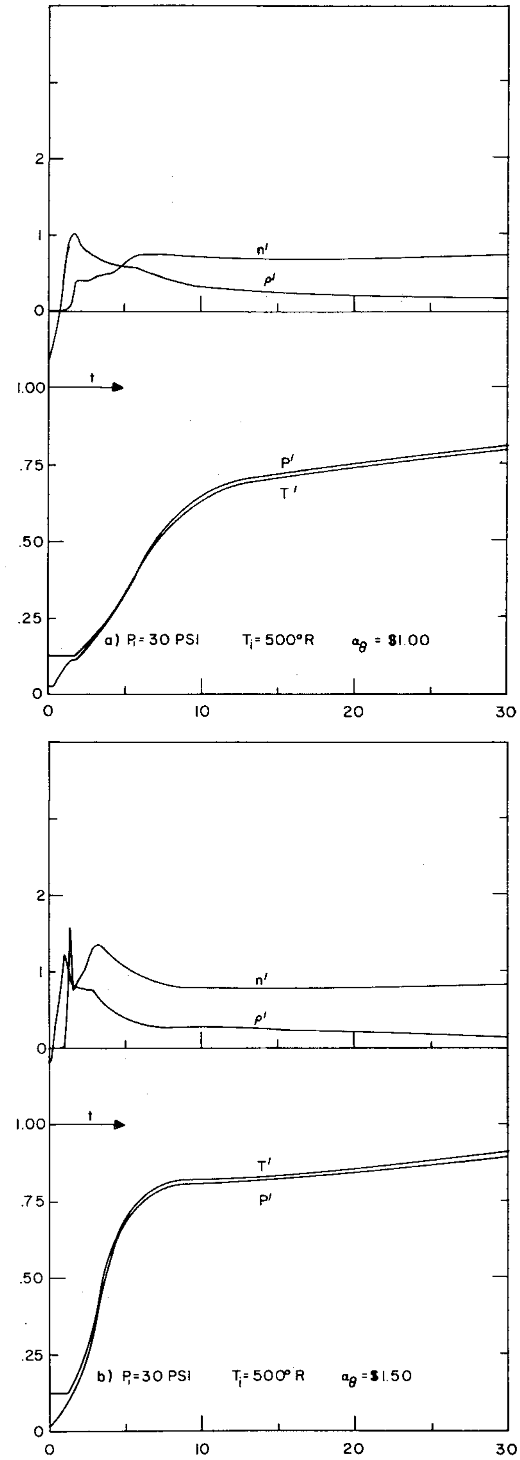

Fig. 10 Open-loop response of the system variables for space operation with the reactor initially subcritical

easily, and a parametric evaluation of its importance is essential.

A value of \$2.20 is taken as the basic value for α_H,[‖] and Figs. 8a and 8b, with P_1 equal to 30 psi, show open-loop critical runs with α_H equal to \$1.50 and \$2.20. The comparison of these runs is complicated by the fact that the final control-rod reactivity must change as α_H changes in order for the total reactivity to become zero for the desired steady-state values of temperature and pressure. If α_H is increased, more control-rod reactivity must be inserted in order to control the system. In Figs. 8a and 8b the control-rod ramp speed was kept fixed at 0.5 β/sec, whereas the final value of ρ_{cr} was adjusted according to the value of α_H.

In spite of this complication, it can be seen that the variation in α_H altered the height of the neutron density and temperature pulses but had no other significant effect. These changes can be analyzed by means of the changes in total reactivity ρ'. As α_H is decreased from \$2.20 to \$1.50, the maximum in ρ' is decreased. The reduced maximum value of ρ' is due mainly to the change in α_H, because the speed of the negative reactivity feedback depends on the reactivity coefficients. The resultant changes in the initial neutron density and temperature responses naturally follow from this reduction in the reactivity peak, since the long-term response continues to be limited by the long reactor period.

[‖] The value of the coefficients α_H and α_θ is often given in "dollar units," where \$1.00 of reactivity is equivalent to 1β.

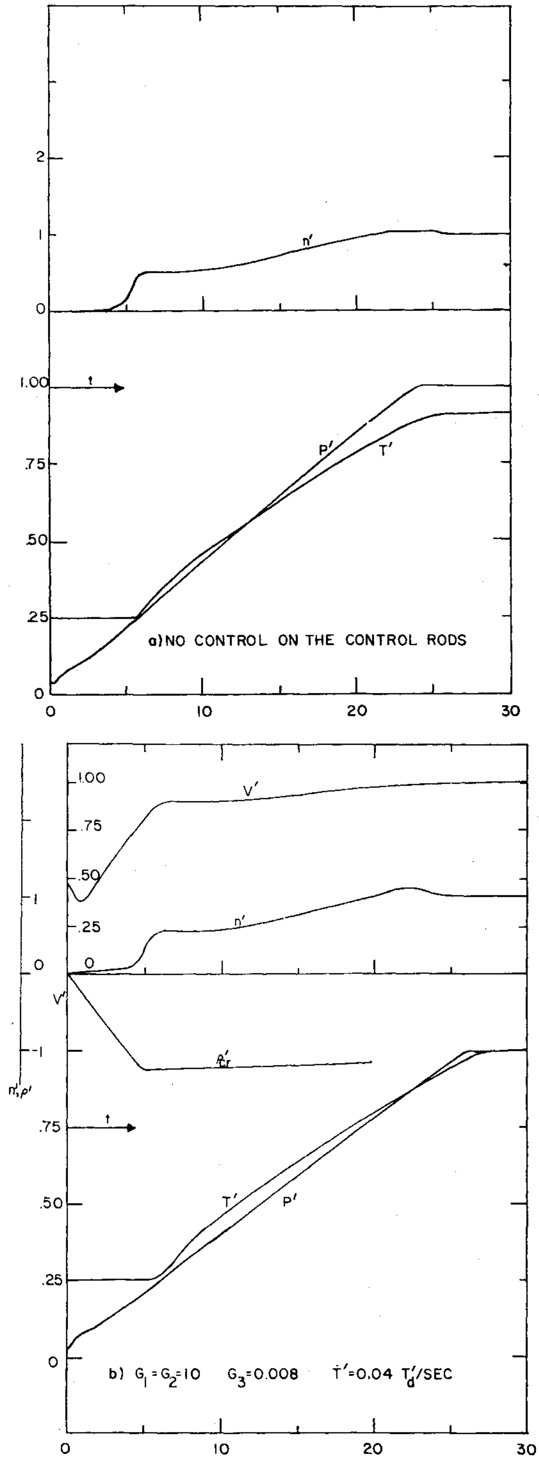

Fig. 11 Controlled response of the system variables for space operation

Bare Core Temperature Coefficient α_θ as a Parameter

A negative temperature coefficient α_θ insures open loop stability for the rocket system under investigation in this study.[6,7] The magnitude of this coefficient is of importance in the problems of control and for subcritical starts. As in the case of α_H, the calculation of this coefficient is very difficult. Consequently, a value of \$1.00 from room to design temperature is assumed for the basic α_θ, and variations about this value are studied for their effect on the overall response.

Figure 9a, ρ/β/sec of 0.5/sec, can be considered along with Fig. 8b, which is the basic case for a 0.5/sec reactivity ramp speed (i.e., α_θ = \$1.00 and α_H = \$2.20). Here again a problem of interpretation arises because of the variations in control-rod reactivity with α_θ. In this case, the problem of isolating the influence of α_θ is even more complex because the value of α_θ determines the initial value of ρ_{cr}' as well as contributing to its final value.

It would be expected that a decrease in α_θ, for example, would result in a greater initial power rise and a resultant increased initial temperature response. This effect is shown by comparing Figs. 8b and 9a. In these cases, in spite of the smaller initial value of ρ_{cr}' and its more negative final value, the effect of the decrease in α_θ is less built-in control and greater initial peaks in neutron density and temperature.

Figure 9b shows the effect of a small swing in ρ_{cr}' between two large, positive values. Here there is a reversal of the trend, in that the neutron density peaks very sharply at a value substantially greater than the value in Fig. 9a, which is for a smaller α_θ. In this case, the positive contribution to reactivity of control rods, which compensates for the increased negative temperature effect, is completed before the initial transient in reactivity is finished. Thus, the reactor goes prompt critical due to the initial hydrogen flow and before the negative temperature coefficient becomes operable.

It should be noted, in considering the influence of both α_θ and α_H, that variations in these coefficients really have very little effect on the behavior of the dynamic system. No unforeseen complications arise if these coefficients are varied as much as 50% from the values chosen for the basic system, assuming that enough control-rod reactivity is available to compensate for steady-state contributions from hydrogen density and core temperature and that a ramp speed of 0.5β/sec is attainable.

Other Parametric Variations

Other less important factors were considered parametrically in order to ascertain their effect, if any, on the overall response. Since these factors were found to contribute generally negligible changes to the system dynamics, no figures are included for their elucidation.

The basic design considered in this study included a three-stage turbine. The effect of a one-stage turbine is investigated. Since the moment of inertia of the one-stage turbine is approximately one half that of the three-stage turbine for representative turbine designs, the variables N' and P_4' experience a somewhat greater initial rate of increase, but the effect is small.

The neutron lifetime l is varied about the value 10^{-4} sec with very little effect noticeable on T_4' and P_4' but with some changes visible in n' and ρ'. Reasonable variations in the constants β_1, β_2, β, λ_1, and λ_2 were made, with little change appearing in the overall system response.

This study has shown that, in general, the open-loop, critical, start-up response is inherently well behaved. Also, the fact that temperature is an integration away from power, so to speak, makes it possible to tolerate very large power excursions without greatly exceeding design temperature limitations. The reactor, therefore, can be set on a prompt critical period without any categorical objections due to temperature limitations, although temperature rise rates may be excessive.

Subcritical, Open-Loop Response

During these starts there is no control-rod movement. The control rods are fixed at a position such that at the design condition the resulting reactivity from control rods, bare-core temperature coefficient, and hydrogen density coefficient is zero, and the reactor is just critical at that condition. Consequently, the position of the control rods makes the reactor subcritical before starting. The level of subcriticality is determined by the final temperature and pressure as well as the bare-core temperature and hydrogen coefficients.

Figures 10a and 10b depict some of the results of the analog computer study for the subcritical, open-loop system. Here again the "basic" system is defined by the constants and design values of the variables listed previously. The leak term L' of Eq. (11) is considered to be negligible, and the initial temperature T_4 is 500°R. Roughly 15 decades in neutron density must be covered in going from spontaneous fission power to full power in a reactor of this type (or 10 decades from a reasonable source power). A factor of only 10^5 is possible on the analog computer. It was tacitly assumed, therefore, that the reactor automatically would cover the initial 5 or 10 decades in neutron density in a time that could be neglected compared with the other transients of the system. It has been shown that the validity of this assumption depends upon the ability of the reactor to go prompt critical. It can be shown that prompt criticality is insured if α_θ is greater than β.

System Response

Figure 10a can be considered as the basic response because $\alpha_H = \$2.20$, $\alpha_\theta = \$1.00$, $P_1 = 30$ psi, and $T_4 = 500°R$ at time equal to zero. The response is very similar to the basic response of the critical reactor (see Fig. 8b). There is a delay of a few seconds in the initial temperature and pressure pulses, but the final time behavior is identical with the critical start case. The peak in neutron density is decreased appreciably, although the reactivity maximum is somewhat increased, reaching a value greater than β in this subcritical start.

Parametric Variations

In Figs. 10a and 10b, reactivity exceeds a value of β during the initial rise as can be predicted from the respective α_θ values. The response in Fig. 10a is somewhat slower than that in Fig. 10b, and the initial neutron density pulse is lowered appreciably. Although not shown, there are very few changes in the response over the entire variation of α_H from \$1.50 to \$2.50. Also, the effect of the variation in τ_r is negligible on the characteristics of the start-up process. The slight effect on the initial transient is as predicted from the critical, open-loop results.

Analysis of the Controlled Nuclear Rocket

Limitation of the Uncontrolled System

If the propulsion system is uncontrolled, the results of this study show that the variables reach 80% of their design values in approximately 15 sec. Design conditions then are approached in a much slower fashion, roughly 80 sec being required to reach steady state. This is due to the long half-lives of some of the precursor groups, which limit the rate of increase of neutron density. With an appropriate control-rod reactivity ramp, the temperature and pressure attain their design value within the first 5 sec. However, after going through a maximum at their design values, the temperature and pressure decrease and then increase slowly, reaching steady-state values for a second time in appoximately 80 sec. In most cases, the initial transients are governed by the turbomachinery; the acceleration of the pump speed governs the rate of increase of the hydrogen pressure, which in turn delivers a large amount of positive reactivity to the reactor. The longer transients are dictated by the reactor. The long-term transients are undesirable because a large quantity of propellant will be used ineffectively. On the other hand, a too rapid rise of temperature in the initial period may pose severe thermal shock problems. This suggests that a control system should have the following functions: 1) limit the temperature gradient to some allowable value (e.g., 0.025 T_d/sec); 2) eliminate the long temperature transients and bring the temperature to its design value as soon as possible within the limits mentioned in 1; and 3) eliminate possible perturbations from the design conditions and make corrections for changes in thrust and specific impulse while the engine is in operation.

Proposed Control Mechanism

The control mechanism should be a stable system with rapid response, and it should allow very little overshoot. An overshoot in temperature is especially dangerous, as the reactor is designed to operate near the limiting temperature to obtain maximum specific impulse.[8]

A limiting factor in this control scheme is the speed at which the control rods can be moved. This may not be important for corrections and perturbations of the system at steady state. However, control of the temperature transients requires fast moving control rods, and the limitation on the speed of control rods is one of the factors that make accurate control during start-up very difficult in the present system. For this reason, other ways were considered to control the temperature gradient, and the following paragraphs describe the method chosen.

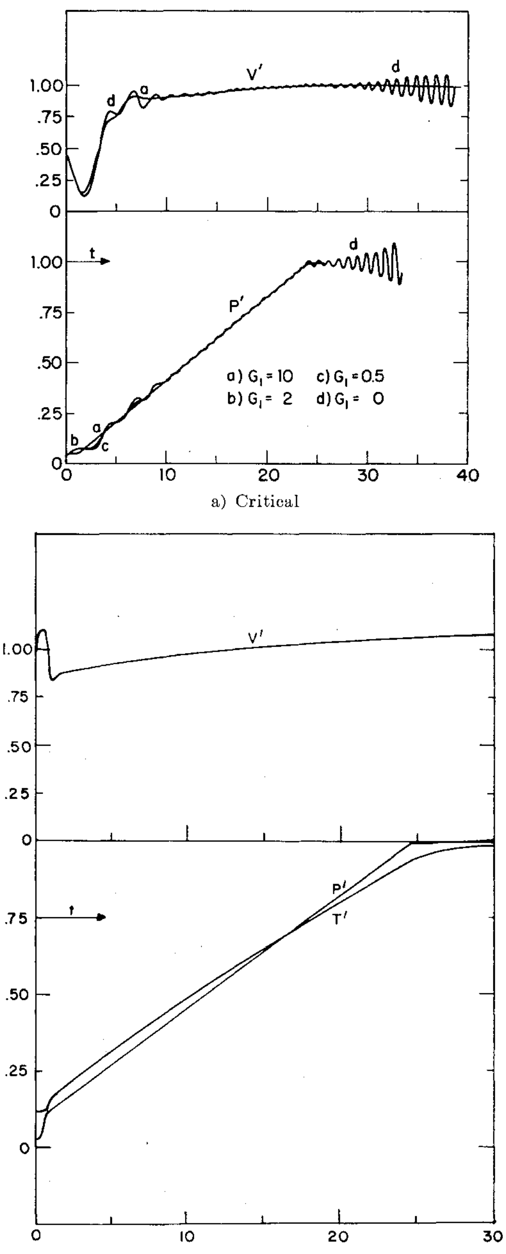

Fig. 12 Controlled response of the system variables for space operation

From the results of the open-loop system, it can be concluded that pressure and temperature experience approximately the same transients during the starting period. This suggests that, by controlling the core exit pressure P_4' during the starting period, it might be possible to have the temperature follow the desired transient.

The control system regulating the system pressure employs a valve that closes if there is an undesired increase in pressure so that the turbine delivers less torque to the pump. Thus the pressure decreases to its desired value. The control system also will bring the pressure back to the desired value when there is an undesired decrease in pressure. The ratio of the pressure after the valve to the pressure before the valve is a function of only the valve area. This ratio is set equal to a fictitious valve setting,

$$P_6'/P_5' = V'(A) \tag{27}$$

Assuming $P_5' = P_4'$, then

$$V' = P_6'/P_4' \tag{28}$$

The valve is controlled by a proportional plus integral controller according to the equation

$$V_x' = G_1(P_D' - P_4') + G_2 \int_{t=0}^{t} (P_D' - P_4')dt + V_i' \tag{29}$$

Proportional control dominates when the error $(P_D' - P_4')$ is large. If only proportional control is used, a steady-state error exists if the initial and final valve settings are not the same. The integral control then is required in order to make the error at steady state go to zero. During the starting operation, this desired pressure should change in such a fashion that the resultant temperature gradients are within the desired limits.

To simulate the control mechanism, a first-order lag is installed between the desired valve setting V_x and the actual valve setting. Therefore,

$$V' = \frac{1}{\tau_v} \int_{t=0}^{t} (V_x' - V')dt + V_i' \tag{30}$$

The range of the valve setting V is between 0 and 1 for normal starts. It may be necessary to increase thrust 10 to 20% over the design thrust, and the turbine should be overdesigned to allow for this larger thrust. The thrust can be regulated by controlling the desired pressure P_D'.

The control rods are controlled only by an integral controller. When a thermocouple is used to measure temperature, the following equation describes this control with the initial value of ρ_x' equal to zero:

$$\rho_x' = G_3 \int_{t=0}^{t} (T_D' - T_4')dt \tag{31}$$

When the temperature is calculated from pressure and mass rate of flow measurement by $T_4'^{1/2} = P_4'/\omega_4'$, the following equation is used:

$$\rho_x' = G_3 \int_{t=0}^{t} (T_D'^{1/2} - T_4'^{1/2})dt \tag{32}$$

Again, in order to simulate the physical apparatus, a first-order lag is introduced:

$$\rho_{cr}' = \frac{1}{\tau_{cr}} \int_{t=0}^{t} [(\rho_x' + \rho_{\text{ramp}}') - \rho_{cr}']dt + \rho_{cri}' \tag{33}$$

Rather than making the desired temperature T_D' a function of time, it was decided to avoid more complications by keeping T_D' constant and equal to the design value during the starting period. For a critical start, most of the change in reactivity from initial to design value is achieved by means of a control-rod reactivity ramp instead of by the temperature feedback controller. Although the control-rod reactivity ramp and the temperature feedback controller operate independently, they must be designed together for optimum operation.

In this program the amount of reactivity of the control-rod ramp was changed in such a way that, at the end of the starting period ($P_4' = P_D'$), the total amount of reactivity from the control rods was (x) dollars. In that case the control-rod ramp should gave $[(x) - \rho_x'/\beta]$ dollars of reactivity.

Results for the Controlled System

Controlled, Critical System Response

If the loop is closed with a valve controller, the core-exit pressure P_4' follows the desired response, but the temperature, after following the desired transient response, reaches steady state much later than the pressure. Figure 11a shows this behavior and should be compared to corresponding open-loop responses. Figure 11b shows the system response when both the valve controller and the control-rod feedback controller are introduced. The temperature then reaches its steady-state value a few seconds after the pressure. The values for the gains G_1, G_2, and G_3 in Eqs. (29) and (31) are adjusted so that the best response is obtained. When $G_1 = 10$, $G_2 = 10$, and $G_3 = 0.008$, the temperature gradient stays below 0.04 T_d/sec, and the long-term transients disappear.

Figure 11b also shows the valve response V' and the combined reactivity of the control-rod reactivity ramp and the control-rod reactivity contribution from the temperature feedback control. The valve setting V initially is equal to 0.44 V_d and reaches its design value at the same time that the pressure attains its design value.

An examination of results for various desired temperature gradients shows that the pressure gradient controls the temperature gradient to a great extent. The controlled system behaves well for control-rod ramp speeds from 0.1 to 0.5β/sec.

In Fig. 12a, the proportional gain G_1 is changed from 10 to 0. The starting transient pressure responses for these G_1 values are almost identical. However, when $G_1 = 0$ and $G_2 = 10$, the system is unstable for perturbations at the design point. A study of the influence of the integral controller gain G_2 on the system response indicates that as the gain G_2 decreases the decay of the error becomes slower. When G_2 is equal to zero and the integrator is removed, the steady-state error is no longer zero.

The system response is not very much influenced by different initial valve settings. The ideal response would be obtained if the initial valve setting were set exactly at that value resulting in a perfect pressure gradient (i.e., P_4' follows the desired ramp exactly). Also, the system behavior is not greatly influenced by changing τ_V from 0.1 to 5 sec. For a larger time constant of $\tau_{cr} = 5$ sec, the temperature rises faster initially due to the lack of negative reactivity from the control rods.

From these results it can be concluded that the open-loop critical system is controlled easily when a controlled temperature gradient of 0.08 T_d/sec or less is desired. The control of temperature gradients greater than 0.08 T_d/sec is more difficult. However, larger temperature gradients probably will not be allowed, due to the resulting severe thermal shock problems. Also, the long temperature gradient can be eliminated by using a small gain on the temperature feedback controller on the control rods.

Controlled Subcritical System Response

The subcritical system is controlled in the same manner as the critical system. Results show that the temperature does not follow the desired transient response completely to the design point. This is because the pressure does not follow the desired gradient. In Fig. 12b, the turbine is 10% overdesigned. In this case there is enough torque available to allow the pressure to follow its desired gradient. Temperature again follows its desired response quite well. The same gains on the controllers are used as for the controlled system for critical starts ($G_1 = 10$, $G_2 = 10$, $G_3 = 0.008$). For these subcritical starts the initial control-rod settings are not equal to their design values but are slightly lower. This reduction in control-rod reactivity is cancelled by the integral feedback controller of the control rods after the transients have been completed. The performance of the controlled, initially subcritical system is then very similar to that of the controlled critical system.

Controlled Change in Design Point Operation

For sudden increases of the desired pressure, when the system is in operation, an excursion of the power level and consequently a change in temperature can be expected. The turbine must be overdesigned to bring the pressure a certain percentage over its design value. The system behavior for a change in desired pressure shows that the resultant temperature excursion is a much smaller percentage of its design point value than the percentage change in design pressure. For example, a 10% increase in pressure results in a temperature excursion of 2.5%, whereas a 20% increase in pressure results in a temperature excursion of 4%.

References

[1] Mohler, R. R. and Perry, J. E., Jr., "Nuclear rocket engine control," Nucleonics **19**, 80 (April 1961).

[2] Hess, G. K., Jr., Demuth, H. B., Brown, E. A., and Mohler, R. R., "Control systems for the KIWI-A nuclear reactor rocket engines," Inst. Radio Engrs. International Symposium, Las Vegas, Nev. (October 25, 1961).

[3] "Starting and control characteristics of nuclear rocket engines," Northern Research and Engineering Corp. Rept. 1034-2 (February 1961).

[4] Yoshitani, Y., "The thermal performance of homogeneous graphite reactors," Mass. Inst. Tech. S. M. Thesis (June 1960).

[5] Felix, B. R., "Dynamic analysis of a nuclear rocket engine system," ARS J. **29**, 853–862 (1959).

[6] Smith, H. P., "Closed loop dynamics of nuclear rocket engineering with bleed turbine drive," Nucl. Sci. Eng. (to be published).

[7] Smith, H. P. and Stenning, A. H., "Dynamics and control of nuclear rocket engines," Nuclear Eng. Dept., Mass. Inst. Tech., Contract DSR8246, performed for Pratt & Whitney Div., United Aircraft Corp. (August 1960).

[8] Schultz, M. A., *Control of Nuclear Reactors and Power Plants* (McGraw-Hill Book Co. Inc., New York, 1955), Chap. 11.

MARCH 1963 AIAA JOURNAL VOL. 1, NO. 3

Liquid Injection Thrust Vector Control

C. J. Green* and Foy McCullough Jr.†

U. S. Naval Ordnance Test Station, China Lake, Calif.

The technique of obtaining thrust vector control by the injection of liquid into the supersonic region of a rocket nozzle has been studied. This paper presents the experimental results obtained with various liquid injectants together with the effects of some of the more critical physical parameters. Liquids studied were water, Freon-12, perchloroethylene, nitrogen tetroxide, and bromine. In addition, unsymmetrical dimethylhydrazine and inhibited red fuming nitric acid were injected simultaneously to explore the effect of energy release in the nozzle exit cone with bipropellant injection. Data on the relationships of side force to injectant flow rate, the effect of axial location of the injection port, the effect of injection pressure, and the effects of injection properties are presented and discussed.

Nomenclature

A = injection port area, in.^{2}
F_m = main thrust, lb
F_s = side thrust, lb
g = gravitational constant, ft/sec^2
I_{sp} = specific impulse (motor), sec
$\dot{W}_c$ = main propellant flow rate, lb/sec
$\dot{W}_s$ = secondary flow rate, lb/sec
ρ = injectant density, lb/ft^3
V_s = injectant velocity, fps
C_V = velocity coefficient
Δ_p = injection pressure, psi

Received by ARS March 9, 1962; revision received December 10, 1962.

* Aerospace Engineer (Propulsion and Power), Test and Evaluation Division, Propulsion Development Department.

† Head, Test and Evaluation Division, Propulsion Development Department. Member AIAA.

As the performance of propulsion systems increases, accompanied in most cases by increased combustion temperatures, the problem of satisfactory materials for mechanical thrust-vector-control devices becomes increasingly critical. With this problem in mind, a study of possible techniques of thrust vector control was initiated at the Naval Ordnance Test Station in 1958.[1] The technique of injecting a fluid into the expansion cone of a supersonic nozzle (secondary injection) to produce a usable side force or thrust vector force appeared to be the most promising new technique (Fig. 1).

The initial studies of secondary injection[2–6] used gases as the injected fluids. These studies, in general, indicated that for maximum effectiveness the injected gas should be near the combustion temperature of the mainstream gases. To avoid the high temperature materials problems and to facilitate system design and development, the use of liquids as injectants was proposed. The feasibility of using liquids was demonstrated early in 1959.[7]